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# Developing understanding of structural implications of dynamic loading in engineering students through the egg drop contest – theory and measurements

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An egg drop contest recently held at the University of Witwatersrand served to test engineering students' ability to solve non-standard problems. In this contest, students designed and built structures to protect their cargo – a raw egg. The structures were dropped 15 m onto a hard surface. The goal was to prevent damage to the egg upon impact. This paper explores some of the experimental and theoretical techniques that could be used to analyse and design a suitable impact resistant structure. The accelerations of the egg during free-fall and impact were measured by a miniature custom built accelerometer sensor and recorded by a data acquisition system. Measurements show that an egg housed in a test structure and dropped from only 3 m can produce accelerations greater than 70 g. The results from a theoretical model of impact presented in this paper closely matches the measured accelerations. The model predicts that if the test structure investigated would be dropped from 15 m, the egg would be subject to accelerations of approximately 100 g.

## INTRODUCTION

Recently the Civil and Environmental Engineering department at the University of the Witwatersrand held an egg drop contest in which structures housing raw eggs were dropped 15 m onto a hard surface. The aim of the contest was for the structure to protect their cargo so that the egg would survive the drop. The structures were judged not only whether they protected the eggs, but also on their weight and volume; that is, the structures were compared based on a proxy to their cost.

This contest is a problem in structural engineering, where the structure and egg are subject to impact loading. Impact is a type of dynamic loading that could be complicated to model experimentally and to analyse theoretically. In general dynamic loading can produce deflections, and hence stresses, in structures that can be significantly greater than equivalent static loads. Engineering students are more familiar with static or quasi-static loading conditions; the concept of a load that varies in time, resonance phenomena and the fact that structures have natural frequencies might not be fully appreciated. The egg drop contest provides valuable practical exposure to dynamic loading, and the testing gives an experimental demonstration of dynamic load's magnitude and severity.

The aim of this paper is to report on the contest and to present some dynamic loading aspects of the egg drop problem.

Egg drop contests, in various forms, have long been held all around the world. (For a few references see for example http://competitions.asme.org/eggdrop/, www.cwru.edu/events/eweek/eggdrop/ and http://members.tripod.com/mrlewis-classroom/eggdrop.htm.) In general, most competitions allow parachutes and other aerodynamic devices that slow the decent of the egg. One of the rules in the present contest was that no specific design with aerodynamic retardation was allowed. This required the careful consideration of structural and material concepts, instead of aerodynamic design. With a 15 m drop, and no air deceleration allowed, the present egg drop contest is a deceptively difficult engineering problem.

The paper is structured as follows. First the contest rules are described. Then several engineering structures that were entered are presented. To determine the magnitudes of accelerations experienced by the egg, a small scale, custom built sensor and data acquisition system was constructed and fully tested. Next, a simplified theoretical analysis is performed. The experimental results are then compared with the theoretical model results.

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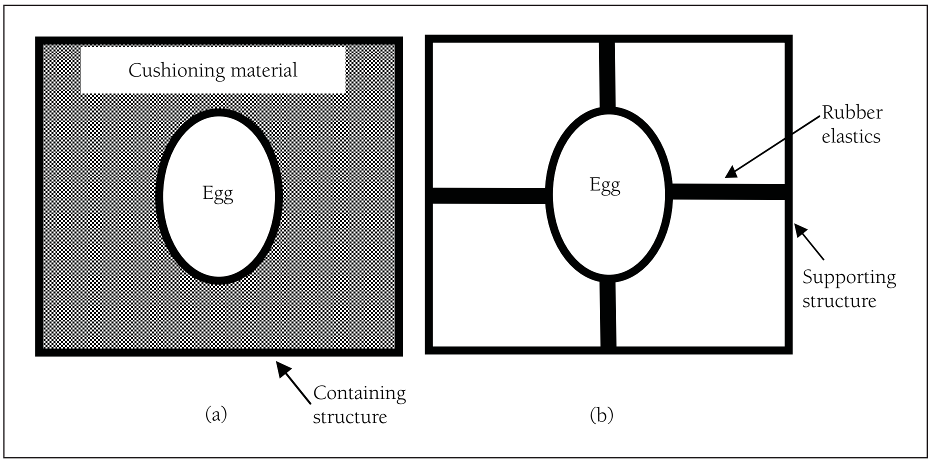

*Figure 1 (a) Egg encased in a cushioning material; (b) egg suspended by rubber elastics*

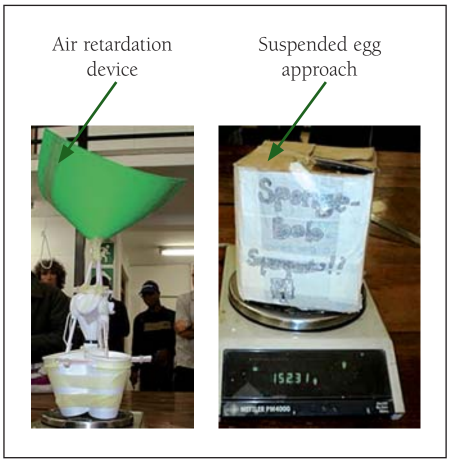

*Figure 3 The two winning structures*

*Figure 2 Photographs of structures following the (a) cushioning material approach; (b) suspended egg approach*

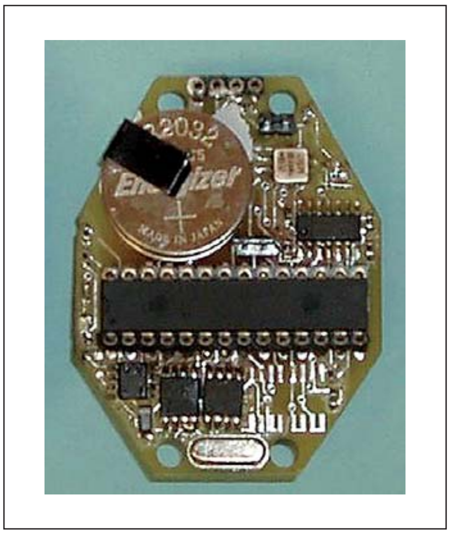

*Figure 4 Sensor board containing accelerometer, microprocessor and memory*

Please note that in this paper the engineering details are specifically omitted, since we plan to conduct this contest periodically in the future.

## THE CONTEST

The egg drop contest was a voluntary project open to anyone who wished to enter. It was specifically not assessed for marks, and the entrants could design and build their structures based on analysis, experiments and/ or intuition. The problem was open ended, *i.e.* no solution concept, material or method of building was suggested or favoured. The entries had to conform only to the contest's rules.

### Rules

The structure housing the egg was to be dropped 15 m off the University of the Witwatersrand Civil and Environmental Engineering Building and land on the tarmac below.

The main rule was that aerodynamic retardation of the free fall of the structure and egg was not allowed. Thus parachutes, air brakes, and gliding/ winged mechanisms and animals were prohibited.

The other rules were:

- The structure had to be deployable. Eggs were handed to the contestants two hours before they were to be dropped
- After the drop, the eggs had to be easily removed from the structure
- No tampering with the eggs was allowed, for example hard-boiling, chemically altering the eggs, etc
- The structure had to land within a 5 x 10 m area below the drop point
- The structures were suspended on a cantilever. A maximum 10 cm string could be used as the suspension cord to centre and level the structure

### Solutions

There were thirteen entries into the contest. The contestants ranged from first to fourth year engineering students. Although all entries were different, especially in the detailing, in the main only two structural philosophies were pursued:

- The egg was surrounded by a continuous or discreet material that would cushion the fall. This material was encased in a container (see figure 1a)

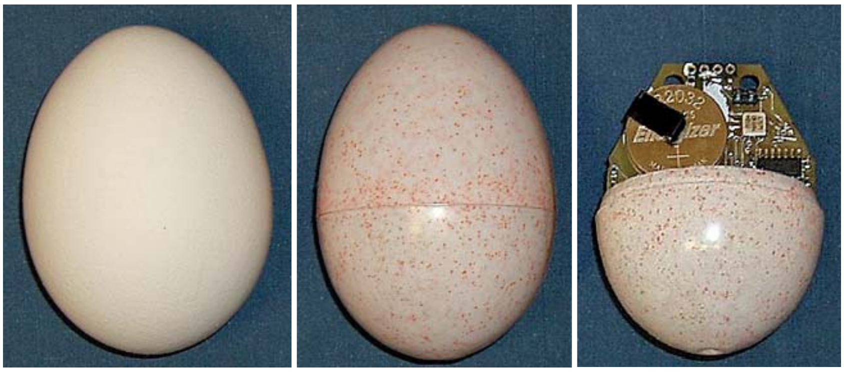

*Figure 5 A real egg on the left; a plastic toy egg in the middle; the plastic egg open showing the sensor system inside*

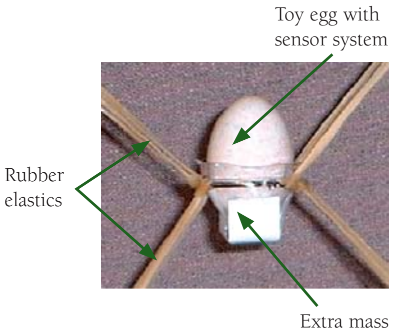

*Figure 6 Detail of plastic egg containing the accelerometer sensor system suspended by rubber elastics*

- The egg was suspended within a structure by rubber elastics (see figure 1b). The elastics were pre-tensioned to ensure that they provide a stiffness at all times

Figure 2 shows four entries into the contest; two examples from the two types of structures. Please note that in general structures were not latticed and the inside details of how the egg was protected could not be seen.

The structures' mass, including the egg payload, ranged from 81,6 grams to 718,3 grams. The average mass was 196,3 grams.

### Results

Of the 13 structures entered, only two eggs survived the fall. There was a wide range of damage in the remaining eggs; ranging from hairline cracks to full pulverisation.

Although all structures started off suspended from a cantilever, the slight breeze ensured that each structure started its fall from a slightly different orientation. This resulted in each structure hitting the ground at different angles. This had a great effect on structures that were not symmetric.

The two structures that protected their eggs are shown in figure 3. The one structure employed the egg suspension technique. The second structure had a conical construct (shown in green in the photograph) at the top. This conical construct, although small in area, retarded the fall of the structure to such extent that the deceleration was very visible. Although not conforming to the rules, the structure with the conical retarding mechanism was not disqualified due to its unique energy dissipating mechanism.

The small number of payload survivors showed that the egg drop problem appeared deceptively simple. The remainder of the paper investigates the magnitude of forces on the egg that can arise during a drop. Only the second popular approach, that of the egg suspended on rubber elastics, is considered.

## MEASUREMENTS

This section presents the experiments that were conducted to measure the accelerations that develop during a drop test. First the developed sensor system is described briefly. Next the experimental set up is presented. The measured results are then shown. A further discussion of experimental results is presented after the mathematical models' results are presented in the next section.

### Sensor system

To determine the forces acting on the egg, the accelerations must be measured. Commercial accelerometer systems (eg those manufactured by CrossBow Technology Inc http://www.xbow.com/ or MicroStrain Inc. http://www.microstrain.com/) could not be used for a variety of reasons: accelerometers were too bulky, required external power, were too slow in acquiring measurements, range of accelerations was too low, or were not standalone. Thus a custom accelerometer sensor system was designed, developed and built. The sensor system was self contained, light and small enough to fit into the size of an egg. The system measured and recorded the acceleration data in real time. After the drop test was complete the data was downloaded from the sensor board onto a PC and analysed. Figure 4 shows the sensor board; indication of board size is given by the 20 mm diameter coin lithium cell batteries. Please note that all power requirements are provided by these coin batteries.

Although the board can support a three-axis accelerometer, only a one-axis accelerometer was used in the egg drop test. The accelerometer had a nominal range of -70 to +70 *g* (where *g* is the gravitational acceleration constant 9,81 m/s$^2$). Data was acquired at 1 000 samples per second. For more details on the sensor system the interested reader is referred to Elvin and Elvin (2006).

A thin shell plastic toy egg was obtained, and the sensor system placed inside. Figure 5 shows a real egg on the left, the closed plastic egg containing the sensor board in the middle, and the open toy egg with the sensor board on the right. Please note that the circuit board was designed and built in such a way as to have a tight fit within the plastic egg. The board was further cushioned in the transverse direction by foam material. Hence there was no motion of the board with respect to the plastic egg.

### Experimental set-up

The plastic egg containing the accelerometer sensor system was suspended in a test structure with rubber elastics. The structure was very stiff compared to the elastics. Further details of this test structure are omitted on purpose. Figure 6 shows the egg containing the sensor system suspended by the elastics.

The sensor board and batteries, together with the toy egg, weigh approximately 15 grams. In comparison, eggs are advertised to weigh *no less than* 50 grams. To obtain realistic acceleration results, the masses of the sensor system and a real egg have to be matched. To increase the mass of the experimental payload, coins were taped to the bottom of the toy egg (as shown in figure 6). The resulting payload together with elastic bands and housing (ie, all the components shown in figure 6) had a mass of 72 grams. This was deemed a conservative estimate of the real payload mass.

Six sets of drop tests were conducted from different heights. These were: 0,5, 1,0, 1,5, 2,0, 2,5 and 3 m. At each of these heights the structure was brought horizontal to the ground and dropped. The structure landed on a hard concrete surface in the laboratory. Since only a one-axis accelerometer was used, only those drop tests that produced predominantly vertical motions of the egg were taken as valid. Vertical motion, and vertical bouncing of the structure was achieved only when the structure was initially horizontal. Non-vertical impact often leads to the structure bouncing onto its side.

Several drop tests were made for every recording period. The exact number of drops depended on how quickly each test could be set up and on the amount of storage memory available on the sensor board. (The implemented sensor board had 64 kbytes of recording space.)

*Table 1 Measured maximum egg accelerations dropped from different heights*

| Height (m) | Maximum acceleration (g) | Minimum acceleration (g) |
|---|---|---|
| 0,5 | 34,50 | –34,0 |
| 1,0 | 45,30 | –45,3 |
| 1,5 | 54,70 | –51,7 |
| 2,0 | 60,33 | –62,0 |
| 2,5 | 70,50 | –69,0 |
| 3,0 | 72,50 | –69,5 |

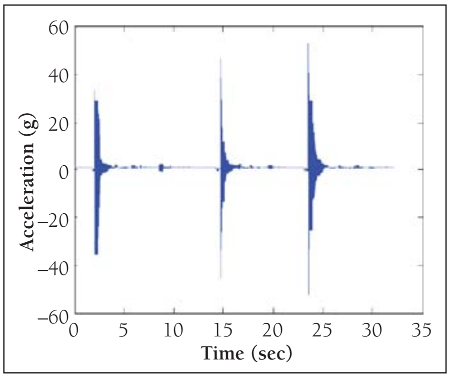

*Figure 7 Typical accelerations of the egg. Three drop tests from various heights were made*

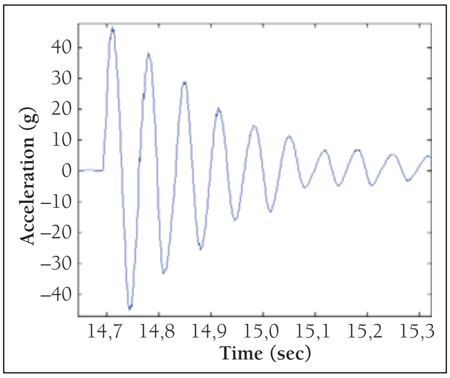

*Figure 8 Acceleration trace of the egg when dropped from a 1 m height*

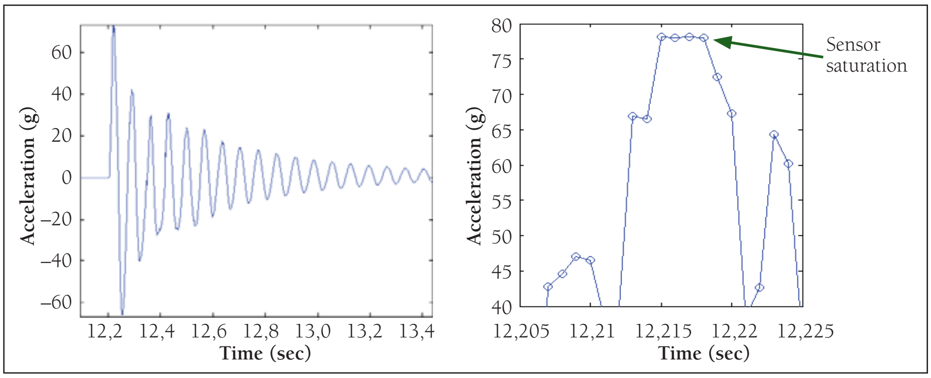

*Figure 9 Acceleration trace of the egg when dropped from a 3 m height. Note the saturation of the sensor when the un-averaged data is considered*

## Results

A typical recorded acceleration trace is shown in figure 7. The figure shows three drop tests in this particular recording sequence. The details of the accelerations experienced by the egg when dropped from a 1 m height can bee seen in figure 8. Please note that slight smoothing, or filtering, is performed by running a 10-point moving average through the data.

All the results were similar to these plots. The only difference was the magnitude of the acceleration. As the height of the drop increased so did the magnitude of the accelerations. For any given height there was some variability between tests. This variability was predominantly caused by the initial conditions (ie how horizontal was the test structure at the start of its fall), and thus how the structure landed.

Figure 9 shows the accelerations recorded from a 3 m drop. Note that this height caused the sensor to exceed the nominal 70 *g* acceleration range for approximately 6 msec. In fact the un-averaged signal saturated at 78 *g* (see inset figure 9). It should be noted that signals greater than the nominal can be measured, however, the accelerometer manufacturer does not guarantee linearity, or validity of these accelerations. Since the nominal limit of the accelerometer used was 70 *g* no tests were conducted from heights greater than 3 m.

Table 1 summarises the maximum accelerations experienced by the toy egg as the structure fell from various heights. Both the positive (first positive peak) and negative (first negative peak) accelerations are tabulated. Drops were conducted at least twice from each height to prove repeatability of the data. The available results are averaged in the table.

Note that saturation beyond the nominal 70 *g* starts to occur from a height of 2,5 m. These values correspond to running average plots. Un-averaged maximum/ minimum results are greater than those in the table.

In summary, the results in figures 8 and 9 show classical under-damped harmonic oscillation. These results lend themselves to theoretical and numerical modelling. In the next section, the measured accelerations will be modelled and a prediction of the expected accelerations for a 15 m drop made.

## ANALYSING THE EGG DROP

### Analytical solution

The egg is considered to be a point mass, *m*, suspended on rubber elastics. These rubber elastics are modelled as springs having a stiffness *k*/2 in parallel with energy dissipating elements or dashpots *c*. This forms a one-degree of freedom visco-elastic system. In this analysis the supporting structure can be assumed rigid since its stiffness is much larger than the stiffness of the rubber elastics. The structure is dropped from a height *h*. The simplified system with rubber bands is shown in figure 10 below.

To simplify the analysis, the structure is assumed not to bounce after impact. Impact occurs at time *t=0*.

For this case the equation of motion is given by the familiar equation:

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad (1)$$

The initial conditions are:

Velocity at impact: $\dot{u}(0) = \sqrt{2hg}$ as calculated from energy balance and (1a)

Acceleration at impact: $\ddot{u}(0) = g$ the gravitational constant of acceleration (1b)

Typically equation (1) is mass normalized to give:

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = 0 \qquad (2)$$

where $\xi = \frac{c}{2m\omega}$ is the damping ratio and $\omega = \sqrt{\frac{k}{m}}$ is the natural frequency of the system measured in radians/second.

Equation (2) is solved using standard techniques (see for example Reference [7]) and yields a solution for the displacement of the egg:

$$u(t) = e^{-\xi\omega t}\left(A\cos\omega_d t + B\sin\omega_d t\right) \qquad (3)$$

Where $\omega_d = \omega\sqrt{1-\xi^2}$ for an under-damped system (*i.e.* where $\zeta < 1$). Constants *A* and *B* are obtained from the initial conditions.

Using the above initial conditions for the egg, and solving two simultaneous equations, it can be shown that:

$$A = -\frac{2\xi\dot{u}(0)}{\omega} - \frac{\ddot{u}(0)}{\omega^2} \text{ and}$$

$$B = \frac{\dot{u}(0)\left(1-2\xi^2\right)}{\omega_d} - \frac{\ddot{u}(0)\xi}{\omega\omega_d} \qquad (4)$$

Thus equation 3 is expressed solely in terms of the damping ratio ($\zeta$), the natural

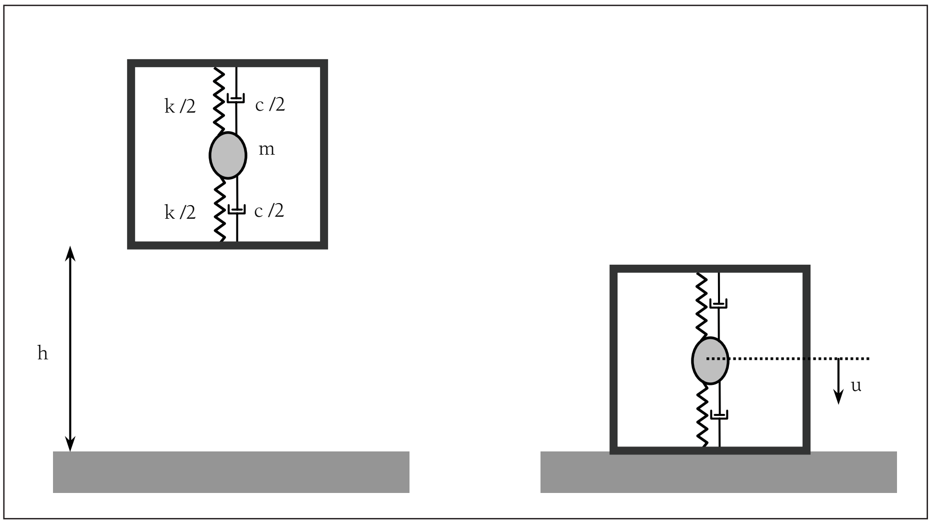

*Figure 10 Schematic of the one degree of freedom model*

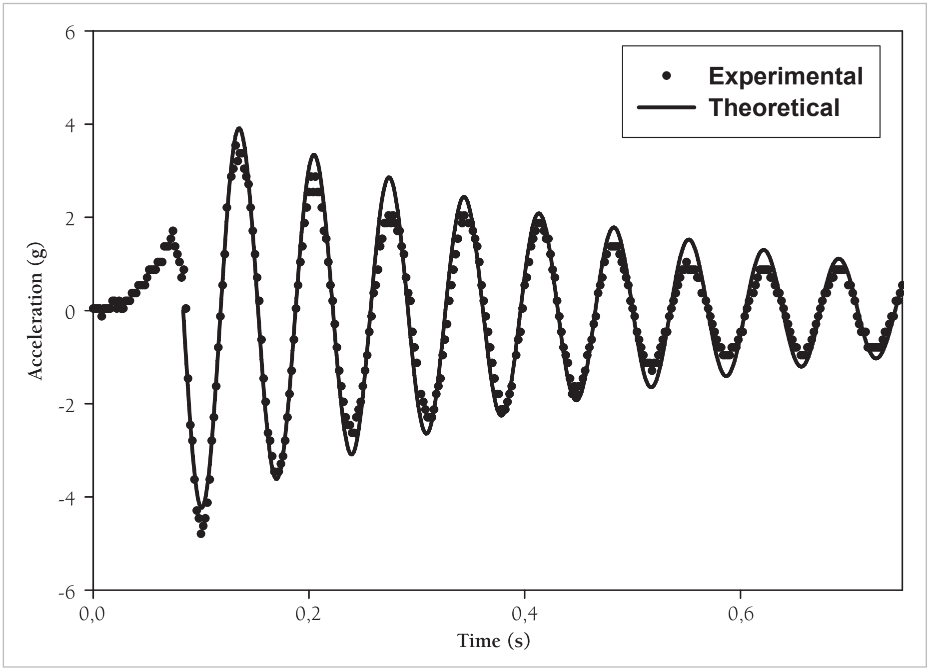

*Figure 11 Egg subject to impulse loading. Experimental data (circles) and best-fit theoretical curve*

frequency (ω) and the initial conditions. The two parameters, ζ and ω, can be calculated by applying an impulse to the egg, measuring the acceleration response and fitting the experimental data. The theoretical damping ratio and natural frequency are chosen to give the best match (in a least-square sense) with the measured impulse response. An impulse loading is used to fit the model parameters since this type of experiment can be well-controlled and small deformations can be applied to the system.

Differentiating equation 3 two times with respect to time gives the acceleration, *a*(*t*):

$$a(t) = e^{-\xi\omega t}\Big((\xi^2\omega^2 - \omega_d^2)(A\cos\omega_d\, t + B\sin\omega_d\, t) \\ + 2\xi\omega\omega_d\, (A\sin\omega_d\, t - B\cos\omega_d\, t)\Big) \qquad (5)$$

The test structure mentioned above was used as the stationary test bed. The egg was displaced with respect to the structure and then released; this provided an impulse loading to the egg. The accelerometer sensor system was used to measure the egg's motion. Figure 11 shows one typical experimental measurement response (circles) and the linear least-squares best-fit (see explanation above) theoretical impulse response (solid line). The calculated natural frequency and damping ratio are $\omega_d$ = 90,48 *rad/sec* (corresponds to a natural frequency of *f* = 14,4*Hz*) and ζ=2,5 %.

The calculated damping ratio and natural frequency in equation 4, together with initial conditions (1a and 1b), were substituted into equation 5 to produce the solution for the acceleration of the egg. The following heights were considered: 1 m, 2 m and 3 m drops. The theoretical and experimentally measured accelerations are plotted in figure 12. Please note that the experimental data is smoothed using a 10-point moving average; this effectively removes bit noise and other non-salient features.

Since the three experiments were conducted separately, the relative position of the peaks is important in figure 12, and not the actual times. For ease of comparison, all three acceleration axes are scaled in the same way.

Note the saturation of the accelerometer beyond the nominal 70 *g* for the 3 m drop test.

## Discussion and comparison

Figure 12 shows a good correlation between experiment and theory. This is especially true at the first peaks. The oscillation frequencies also match well. The theory tends to under predict the measured values (by less than 10 %) at impact, and over predict during subsequent oscillations.

Impact experiments are notorious for lack of control. While drop tests were carefully screened and only predominantly vertical oscillation drop data was accepted, the egg was free to move in three dimensions. In general, due to the impact direction not being perfectly vertical every time, and geometric imperfections in the test structure, the egg did not vibrate only in one direction. The subsequent acceleration trace can be used to determine how vertical the drop was. Figure 12 shows that for the 1 m and 2 m drop heights, all the oscillations are close to sinusoidal – the impact was most probably close to vertical. In contrast, the drop from 3 m produced non-sinusoidal behaviour after the initial impact. The peaks are somewhat flattened and distorted. This could well be due to other modes being coupled into the motion (eg the egg vibrating transversely, or even rotating). Further, nonlinear effects, both material and geometric, that are not considered in the present model, may become important. The focus of this study, the first impact, however, dominates, and since it was in the gravitational direction (the same direction as the accelerometer axis), there is a very good match between experiments and theory.

In this structural configuration, the egg can crack in one of two ways. After impact, the vertical deflections of the egg can become so large that it exceeds the height of the support structure thus hitting the ground upon impact. The deflection of the egg is given by equation 3. The second way the egg can fail is by inertial crushing. The accelerations become so large that the product of the egg's mass times its acceleration exceeds the egg's strength. Instron Inc (www.instron.us/wa/library/streamfile.aspx?doc=86) reports on tests conducted on large hen's eggs; of the eggs tested the maximum point load the egg could withstand is approximately 35 N.

The variation in impact acceleration of an egg with height can be obtained for the drop test using the theory presented above. Utilising the material and structural parameters for the present case, Figure 13 shows the maximum force at impact, given by the

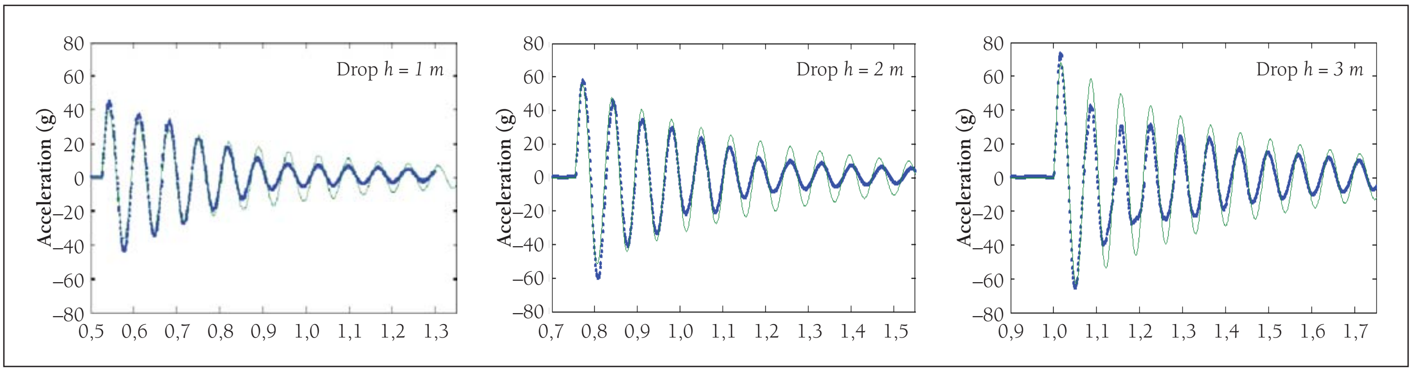

***Figure 12 Acceleration histories for the egg dropped from 1 m, 2 m and 3 m. Circles represent experimental data, solid line is the theoretical solution***

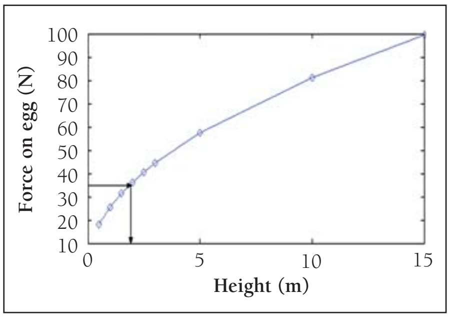

***Figure 13 Theoretical variation in force acting on a 72 g egg dropped from various heights***

theoretical acceleration times 72 grams (the mass of the egg), versus the drop height. The egg's failure point load of 35 *N* is also plotted.

Since the initial velocity (at impact) is proportional to the square root of the height (see equation 1a) and the constants of integration, *A* and *B*, are proportional to the initial velocity (see equation 4), it follows that the acceleration and hence the force will be related to the square root of the drop height – as seen in figure 13. For the test structure, and from figure 13, the egg should fail when dropped from approximately 2 m. There are several significant assumptions. The failure load of 35 *N* depends on the nature of the load applied (point load versus distributed), and on the position of the load on the egg. In the test structure, the supports afforded more of a uniform load on the egg than a point load. Second, small deformation theory is assumed which allows for a closed form solution. Finally, the rubber bands are assumed to behave visco-elastically. In reality rubber is a highly non-linear elastic materials, with a strain-rate dependent response. Taking these assumptions into account, it is expected that an egg would survive a drop from a height greater than 2 m, but the test structure considered would not protect its payload from 15 m.

The large theoretical forces and measured accelerations explain why the egg drop contest is not as simple as it first appears.

## CONCLUSIONS

This paper presented the results from an egg drop contest. To explain the complexity of the structural problem, a custom accelerometer sensor board was built and experiments were performed to measure the impact accelerations. Measurements showed that when the test structure was dropped 3 m, accelerations exceeded 70 *g*. An analytical model was also presented that treated the egg as a one degree of freedom system. The model matched the measured results. The model predicts a parabolic relationship between the drop height and egg acceleration. For the test structure subjected to a 15 m drop, that is, the height in the contest, 100 *g* accelerations are expected.

The egg drop proved to be a challenging problem in structural engineering. It is the intention of the authors to stage similar contests periodically in the future.

## ACKNOWLEDGEMENTS

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