

# Postulates on shear in reinforced concrete

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## COMMENT 1

We read the paper 'Postulates on shear in reinforced concrete' with interest. The author is to be congratulated in helping to bring clarity to what is often an obtuse and complicated area. The paper could serve as a very useful review for undergraduate and postgraduate teaching courses as well. However, we have a few points of discussion:

- Figure 2: In the middle diagram, the 45° diagonal arrows should presumably not be present.
- The Poisson effect for plane stress elements is usually postulated in terms of strains, not stresses. Therefore, the last paragraph under postulate 3 is questionable. (It is acknowledged that under plane strain conditions, transfer stresses can be produced by the Poisson effect, but it remains primarily a strain, that is, deformation effect, rather than a stress effect.)
- The reason for concrete elements such as cubes or columns to fail by cracking in the direction of the principal compressive stress is indeed the Poisson effect. However, it is debatable whether the cracks are caused by true stresses. Rather, there are researchers who hold that concrete fails by a limiting maximum tensile strain criterion, and that consequently the cracking described is a consequence of reaching the limit of the maximum tensile strain. However, we do agree with the sentence that states, 'Ironically, what we call a compressive test is actually a measure of the tensile capacity of concrete.'
- The pure shear rig shown in figure 8 is interesting as it provides an easy method of producing what is commonly considered 'pure shear' stresses in the member. The author states correctly that the mode of failure of this specimen is related to tensile splitting forces. It would be interesting to discuss this in comparison to the usual tensile splitting tests and identify differences, if any.
- The author points out under postulate 5 that true shear failure can occur in concrete where a weak plane exists, for example in composite members. No conclusive reasons or evidence were given for this. Following the author's discussion it may well be that the observed interface failure in such specimens is due to tensile stresses at the interface. The authors of this discussion have carried out extensive interface shear tests on composite

specimens, identifying 'interface failure' to commonly take place in the material close to the interface (0–1 mm) but not at the interface as such, irrespective of interface texture. This indicates possible failure in tension inside the material.

- Reference is made to a 'shear capacity' of concrete. However, the author is at pains to indicate that concrete fails predominantly in tension and that shear forces and stresses are a result of systems of tension and compression. Therefore, is there such a thing as a 'shear capacity' of concrete?

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## COMMENT 2

The author is to be commended for a very interesting and thought-provoking paper.

The paper centres on shear in reinforced concrete beams and slabs. Various mechanisms of cracking in concrete are explained in detail. A number of postulates are defined, which are then substantiated through the use of drawings, finite element analyses and photos of experiments on concrete beams. Further items which are dealt with are enhanced shear strength near supports and shear stresses in slabs. It is interesting to note, as the author points out, that the phrase enhanced shear strength is actually a misnomer because there is usually no variation in beam depth, concrete strength and shear steel closer to the support. In other words, the shear strength is actually uniform throughout the length of a beam.

The author's comments on certain aspects listed below would be appreciated.

- In the introductory paragraph it is stated that our analysis methods are based on linear elastic models but our codes are based on limit state principles. Should it not be *ultimate* limit state principles, because the codes were always based on a limit state, admittedly on an allowable stress in the early codes.
- Reference is made to the elements in figure 1 as differential elements, which presumably has the meaning of very small elements as used in the classical theory of stress analysis. In figure 2, as stated in the text, a tensile force is applied at the corners of the same element. This is impossible, because the corners of a differential element have no area.

■ Even if one is to assume that it is possible to apply forces at the corners of a differential element, the resultant (shear) stress pattern in the third element of figure 2 in effect assumes that the corner forces as indicated in the second element have been *shifted* to the sides of the third element, which resulted in the shear stresses as shown. A study of the compressive and tensile stresses of the third element will show that it is not representative of the force condition in the first element of figure 2.

■ Postulate 1 states that only two fundamental forces exist in structural members – tension and compression. Is the expression ‘fundamental’ intended to be the same as ‘principal’ as used elsewhere in the paper and illustrated in figure 1?

■ Under postulate 1 the statement is made that shear stresses are components of tensile and compressive stresses. Should one not add ‘and vice versa’? The paper itself uses the example of the end of a simply supported beam and notes that at this location the stresses resemble the pure shear case illustrated in the third element of figure 2, which in turn results in (diagonal) tensile and compressive stresses. In other words, in this case the tensile and compressive stresses are components of the shear stresses.

■ With reference to the diagonal crack in figure 4 the statement is made under postulate 4 that because the crack is not vertical or horizontal as shown in figure 5, it is not the result of shear forces. Should it not be stated that the crack is indeed the result of shear forces and its complimentary shear stresses as shown in the third element of figure 2, which in turn resulted in diagonal tensile stresses parallel with the arrows in figure 4, and because the tensile strength of concrete is less than its shear strength, failure is expected to occur along a diagonal plane as shown in the figure, and not along a vertical (shear) plane.

■ When discussing shear stresses in slabs it is stated that the flow of stress within a slab is low compared to that in beams, and the flow of water from a roof is used as an analogy to explain this point, but this correspondent is still baffled. Surely for a slab and beam, both with the same span and type of loading – for example the beam under a point load and the slab under a line load – the flow of stress (axial or shear stress) from mid-span to the supports will be identical.

■ Under the heading ‘Distribution of shears in slabs’ it is stated that for a beam subjected to UDL the shear distribution is linear and triangular, whereas in one- and two-way slabs the distribution is similarly linear but wedge-shaped. This correspondent is at a loss why there is a differ-

ence in the shear distribution of a beam and a one-way slab.  
A truly absorbing paper.

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### COMMENT 3

The author has taken a novel view of the subject, but several of his assertions must be challenged.

### Introduction

At any point in a three-dimensional body, a limitless number of small plane surfaces with differing orientations may be conceived. On each such plane, stresses are acting, and for convenience they may be resolved into their components, one normal to the surface, and two, the shear stresses, acting parallel to the plane at right angles to each other. To fully define the stress condition at the point, the stress components on three mutually perpendicular planes must be given. To determine the stress conditions on any other set of orthogonal planes at the point, the transformation equations may be set up from equilibrium conditions. For a given set of axes normal to these planes, only six independent stress components exist, as shear stresses on adjoining planes occur in pairs, in each of which the two members, one on each plane, are equal. It can be shown that a set of orthogonal axes exists, for which the shear components are zero. These are the principal axes, and the corresponding normal stresses are the principal stresses. The stress condition at a point may also be fully specified by the three principal stresses, and three angles which give the directions of the principal axes: again six independent quantities are required. It is important to distinguish between a force, which is a vector (or first order tensor) and a stress, which is a second order tensor. When referring to a force derived from a stress (by integration over a surface) the orientation of the relative surface must always be clear.

When the values and orientations of the principal stresses have been found, it does not imply that the shear stresses on other planes have disappeared. They are still there and may be significant, as the principal stresses and the planes on which they act are not necessarily the critical ones for fracture or failure

Maximum shear stresses at a point occur on planes orientated at angles of  $45^\circ$  to the directions of the algebraically largest and smallest principal stresses, and are equal to half the algebraic difference of these two principal stresses. Replacing a system of forces acting on a body by its resultant must be done with caution. It is permissible when the equilibrium of the body as a whole

is considered; it is not permissible in the investigation of the deformations and internal stresses in the body.

### Author’s postulates

#### 1 Only two fundamental forces exist in structural members – tensions and compressions

It is not clear what this means. If the author intends internal forces in structures, such as struts, ties or beams, these may be found either from equilibrium of a portion of the structure (considered as a ‘free body’) with the internal forces applied at the selected section, or if the internal stresses are known, from integration of these stresses over the section. Internal forces will then in general include compressive and tensile forces, shear forces, bending moments about two axes, and torsional moments (in the three-dimensional case). Only in special cases will the internal forces consist solely of compressive and tensile forces. I suspect that the author is referring to stresses, in which case his ‘fundamental forces’ could be referring to the principal stresses. On the principal planes, only normal stresses exist. In integrating these stresses over a finite area to obtain forces, one runs into the difficulty that the directions of principal stresses vary with change of position, and their resulting forces over a plane section will in general also contain shear components. It is of course possible to integrate over a surface which is everywhere normal to one of the principal stresses, but such a surface will generally be curved, and the terms ‘tension’ and ‘compression’ then are irrelevant.

#### 2 Shear stresses are components of tensile and compressive stresses applied at an angle to the x and y axes

The author is here confusing stresses and forces. His figure 2 is misleading, in that stress components must always act on a surface, and the correct way of finding the normal stresses relating to shear stresses is to consider an element bounded by the planes on which the shear stresses are acting and the plane on which the normal stresses are acting. Equilibrium conditions are then applied to find the equilibrant. But the important point is that the shear stresses have not gone away; they are still acting on the same planes as before. To ignore the shear stresses is to ignore equilibrium requirements.

#### 3 Two forces will always exist and (are?) orientated orthogonally to one another

This statement is meaningless unless the direction of the forces and the planes on which they are acting are specified. Stresses may be regarded as acting on a point, but internal forces act on finite areas. The author refers to ‘element forces’. These are

individually of infinitesimal size and are only significant when they are considered in conjunction with other such forces, as in the investigation of equilibrium of the element on which they are acting, or when they are integrated over a finite area.

The author has in his presentation included only two-dimensional cases. In analysis, many structures may in fact for convenience be regarded as two-dimensional, but it must be borne in mind that all structures are in fact three-dimensional. This is significant as principal stresses in the direction normal to the two-dimensional plane may be associated with critical shear stresses in the structure, with components in this third direction, even if this normal stress is zero.

The author uses a mechanism consisting of bars and springs to model the Poisson effect. Apart from the fact that this model is a poor representation of a continuous material, I fail to see why the author concludes that it demonstrates that 'a stress applied in one direction will produce an opposite stress at 90 degrees'. Assume a vertical plane cutting through the mechanism, both bars and springs, and consider the horizontal equilibrium of a portion of the mechanism to one side of this plane as a 'free body', with the forces in the bars and springs now considered as external loads. The springs produce horizontal forces in one direction, and the bars equal forces in the opposite direction: there is no resultant. It is easy to demonstrate that no stresses in the direction transverse to uni-axial loading are associated with the Poisson effect. Consider a long bar of elastic material loaded at its ends in tension. The bar will lengthen in the direction of loading and contract uniformly in the transverse directions. There are of course no stresses on the lateral surfaces. Consideration of equilibrium and the stress-strain relations show that there can be no stress components in the directions normal to the long axis of the bar.

#### **4 Only two modes of possible failure exist – a tension and compression**

This is incorrect. It would be much closer to the truth to state: 'Two modes of failure exist – tension and shear.' Failure in the normal sense does not occur in pure compression, when all three principal stresses are compressive and equal and there are consequently no shear stresses. A portion of a rock layer kilometres below the earth's surface is subjected to tremendous compression forces. The material properties may be altered, but the rock continues to support the imposed load; it has nowhere to go. The three principal stresses are all the same, or nearly so, and shear stresses are consequently negligible.

Failure due to compressive force, whether due to direct loading or to bending, is

actually due to shear stresses reaching critical values. I once observed a compressive test on a brittle cast iron cylinder. Failure occurred suddenly with an inclined crack across the whole specimen. I have tested many concrete cubes in compression, and when after failure the shattered material on the exposed surfaces was removed, I found two irregular but well-defined pyramids with sloping faces: clearly shear failure occurred in both cases.

The criteria for failure are treated in the subject of failure theory, and several formulas have been produced, as given in the book *Plasticity in reinforced concrete*, by W F Chen (McGraw-Hill 1982), chapter 5: Failure criteria of concrete. The formulas all give limiting shear stresses.

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### **RESPONSE**

The paper entitled 'Postulates on shear in reinforced concrete' seems to have generated a considerable interest on the subject of shear. Although only the comments from four prominent academics and practitioners have been included, many more have corresponded in recent months. The message is clear – an understanding of shear seems to elude our understanding. The concept of shear is so basic to engineering and material science, yet the debate rages on.

Experience, however, often teaches us another lesson, sometimes conflicting to what is taught. The intention of the paper was to express these views and formally define shear in a set of postulates. Admittedly, some minor 'tweaking' may be required to align the postulates to read correctly. For this reason, comments provided by various academics and practitioners are welcomed and valued.

#### **M G Alexander and H Beushausen's comments**

In figure 2, the middle diagram should have three arrows. The arrow orientated along the diagonal is the stress applied at 45° to the x or y axes. The other two arrows are components of stress. However, to follow convention, a dotted line should extend from the diagonal arrow to each of the components. The correct version of figure 2 is published on the web.

The comment is quite correct that Poisson's ratio is postulated in terms of strains rather than stresses, the reason being that for compressible materials, strains applied in one direction will always produce strains at 90°. The same cannot be said for stresses, unless the material is restrained

orthogonally. In reinforced concrete, however, restraint (or containment) is **almost always present due to the massive nature of RC structures, reinforcement, support conditions, friction and other external factors**. It is the restraint that generates the stress at 90°. Diagonal tension and splitting tensile cracking occur due to the coupling consequence of Poisson's effect and internal restraint. Thus, the postulate is applicable to RC structures. Having said this, I would tend to agree for the sake of correctness and change the postulate to define Poisson's ratio as conventionally read (in terms of strains), with the proviso that stresses will be generated at 90° due to internal restraint. It is the restraint of internal strains that leads to cracking of RC structures.

Alexander and Beushausen suggest that even in the case of pure shear applied to a weak plane (that is, the interface of composite structures), the mode of failure may also be tensile. This is a very interesting concept and has spurred much thought in the possibility.

Alexander and Beushausen also question the validity of the concept of 'shear capacity'. I agree that if the mode of failure is the result of a system of tensions and compressions (as claimed), the concept of shear capacity is irrelevant and substantiates the accusation that conventional theory is fundamentally flawed.

#### **D du Plessis' comments**

The limit state condition referred to is the ultimate limit state. I agree this should have been clarified in the paper.

Du Plessis states that it is impossible to apply a stress to the corners of a differential element, since no area exists. The comment is valid. However, the differential element was left in its original orientation to illustrate a point and for the ease of understanding. The principle, however, remains intact – shear stress is a component of a compressive or tensile stress orientated at an angle to the differential element. One may refer to Mohr's circle for a confirmation of this concept. Furthermore, literature often illustrates a deformed differential element subjected to pure shear. The deformed shape is consistent with a stress applied at the corners of the differential element.

Fundamental forces are referred to as moments, axial and shear stresses. Principal stresses are referred to as stresses along a plane that are maximum or minimum, excluding shear stresses.

Du Plessis suggested that postulate 2 should be expanded to state that not only are shear stresses components of compressive and tensile stresses, but vice versa. The statement is correct.

I do not agree that the diagonal crack in figure 4 should be referred to as a shear

crack simply because shear stresses exist in that region. The crack is along a diagonal and separation is clearly perpendicular to the crack line. The mode of failure implies that the crack is tensile. If the crack was orientated vertically or horizontally and slippage is observed along the crack line, then one may refer to that failure as a shear failure.

The analogy of a slab to the flow of water on a roof is to illustrate that the flow of load per unit width of slab will be less than the flow of load in a beam. Although the amount of imposed load will not change as it flows from the slab to the beam, the load carried by a slab is spread out over a larger area and therefore the shear stresses are less.

When diagramming shear forces in a beam, the shape of the shear diagram is two-dimensional and triangular for a UDL. However, in a slab, the shear diagram should be three-dimensional to illustrate the shape of the shear diagram over the entire

slab. In three dimensions, the shape of the shear diagram in a slab is wedged-shaped.

### **Von Willich's comments**

In the first postulate, the use of the word 'force' may be replaced by the word 'stress'. This seems to be a prevalent comment to make the postulate more correct and to fit into conventional theory. The concept, however, remains intact – all force resultants are composed of tensions and/or compressions.

In the second postulate, the shear stresses are not ignored nor are they considered to have disappeared. The intention of the illustration is to demonstrate what a shear stress is – a stress applied at an angle to the axes. Mohr's circle confirms this statement.

Von Willich is incorrect in trying to relate the model in figure 3 to a physical form. It was never intended to represent an actual strut/tie system. It is merely a model to represent a material response to loading. For example, an axially loaded column is

often represented as a spring, which represents the elastic stiffness. We cannot say that the model is poor simply because columns are not made of springs. Von Willich seems to have misunderstood the purpose of the model.

A statement is made that Poisson's effect does not produce a stress at right angles. Von Willich used a uniaxially loaded bar as an example to illustrate this point. The example is correct for a long thin bar subjected to ideal conditions, but flawed in a sense that reinforced concrete is never in the form of a long bar. Furthermore, the 'real world' is never ideal. Reinforced concrete always comes in bulk and the reinforcement is commonly in the form of a cage. The physical characteristics of reinforced concrete naturally provide containment of strains in the transverse direction and therefore transverse stresses occur from Poisson's effect, which ultimately leads to cracking. The majority of cracking in concrete occurs as a result of this physical phenomenon.